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NEAR RESONANCE POLARIZATION MODULATION (NRPM), A NOVEL METHOD FOR HIGH PRECISION BEAM ENERGY MEASUREMENT IN STORAGE RINGS*

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Abstract

We propose Near Resonance Polarization Modulation (NRPM), a novel method for high-precision beam energy measurement in storage rings. In this technique, a constant-frequency AC dipole is applied near the spin precession frequency, driving the beam spins coherently. The spin tune can be reliably extracted from the time-dependent polarization signal, enabling a very high-precision determination of the beam energy. Its performance has been demonstrated using the Future Circular Collider e+e- (FCC-ee) Z-pole lattice, exploring a range of configurations including AC dipole strengths and initial polarization levels. The method exhibits robustness against lattice imperfections. Compared to the traditional resonant depolarization (RDP) technique, NRPM offers significantly improved precision, greater tolerance to systematic uncertainties, and simplified operational procedures. Beyond the FCC-ee case study, NRPM is broadly applicable to high-precision energy determination in modern storage rings. The superior precision offered by this technique will significantly advance the state-of-the-art in beam energy measurement, with critical applications in high-energy physics and the measurement of fundamental constants.

INTRODUCTION

Accurate measurement of beam energy in storage rings constitutes an essential requirement in contemporary high-energy physics, providing the foundation for high-precision determinations of elementary particle masses and widths. For storage rings operating with polarized beams, the beam energy (E) is conventionally extracted from the measured closed-orbit spin tune (ν_0), defined as the number of spin precessions per turn for particles on the closed orbit. In an ideal, perfectly aligned lattice, ν_0 is related to the average beam energy along the closed orbit by $\nu_0 = a\langle E\rangle/(mc^2)$, where a is the gyromagnetic anomaly and m the particle mass. In the presence of machine imperfections, the closed-orbit spin tune deviates slightly from this simple relation. Equivalently, ν_0 corresponds to the spin precession frequency $\nu_0\, f_{\mathrm{rev}}$, with f_{rev} the revolution frequency. The center-of-mass energy is subsequently derived from this average energy.

Resonant depolarization (RDP) has long been regarded as the gold standard for high-precision beam energy measurements, with notable implementations at large colliders such as the Large Electron-Positron collider (LEP) [1], at smaller colliders like VEPP-4M [2], and at light sources including the Swiss Light Source (SLS) [3]. The technique operates by sweeping the frequency of an external radio-frequency (RF) field across the spin precession frequency of the beam, the frequency at which depolarization occurs reveals the spin tune and thereby the beam energy. The ultimate precision of RDP is fundamentally limited by kinematic broadening arising from the finite beam energy spread, which smears the resonance onset. In addition, premature depolarization occurring prior to reaching the true resonance condition distorts the fitted line shape and constitutes the dominant source of systematic uncertainty in this method.

The physics program of the Future Circular Collider electron-positron (FCC-ee), particularly at the Z pole and the W-pair threshold, demands an energy calibration with systematic uncertainties substantially smaller than those attained at LEP. The current estimate of the systematic uncertainty at the Z pole is 100 keV [4,5], with the ultimate aim of reducing it to the level of the statistical error of 4 keV. This stringent precision goal motivates a thorough examination and validation of existing energy measurement techniques, as well as the exploration of strategies to further enhance their accuracy.

In this paper, we propose a novel energy measurement technique termed Near Resonance Polarization Modulation (NRPM). In contrast to RDP, which drives the system through the spin resonance, NRPM operates in an off-resonance regime and thereby preserves the bulk polarization of the beam. By applying an external RF field at a fixed frequency in the neighborhood of the spin resonance, a coherent modulation of the spin motion is induced, producing a measurable response whose frequency is directly linked to, and exhibits high sensitivity to, the spin tune. This scheme has the potential to surpass current precision and offers a

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route toward quasi non-destructive, rapid, and highly accurate energy monitoring at the FCC-ee and at other storage rings.

METHODOLOGY

The NRPM technique was inspired by polarization oscillations observed in spin tracking simulations as shown in Fig. 1. An AC dipole providing an oscillating horizontal dipole field is installed adjacent to an arc quadrupole in the lattice, serving as the RF depolarizer. When this AC dipole drives the beam at a fixed frequency close to the spin precession frequency, the beam polarization develops an approximately periodic modulation. The effect is systematic and grows more prominent as the frequency of the AC dipole is tuned closer to the spin precession frequency. This pattern points to a coherent modulation of the polarization governed by a single dominant harmonic.

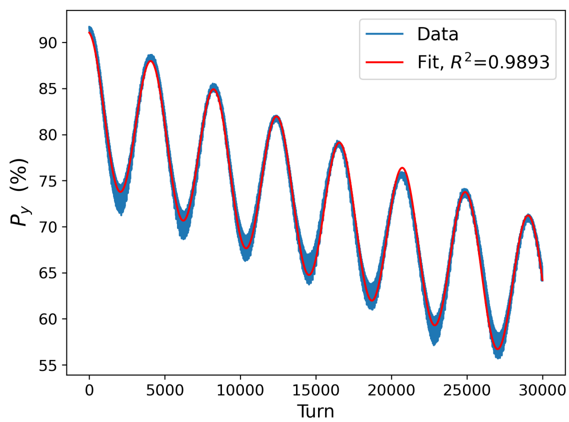

Figure 1: Vertical polarization evolution under application of a fixed AC dipole frequency

$$P(\text{turn}) = Ae^{-\lambda \cdot \text{turn}} + B\sin(\Omega \cdot \text{turn} + \phi) + C \qquad (1)$$

The initial ansatz used to describe the polarization "beat" is given by Eq. (1). The first exponential term represents the decay envelope arising from radiative depolarization, and the second term describes the oscillation. Two characteristic oscillations are present in the system, associated with the intrinsic spin precession and the applied RF field. Through this drive, the spin precession is modulated, producing both a fast oscillation at high frequency, which is largely beyond observational resolution, and a clearly visible slow oscillation. The slow-oscillation frequency is conjectured to be governed by the detuning between the AC dipole frequency and the spin precession frequency, $2\pi(f_{\text{ac}}/f_{\text{rev}} - \nu_0)$.

To validate the proposed analytical form, dedicated Monte Carlo spin tracking simulations were carried out using both the FCC-ee GHC and LCC lattices at the Z energy, with particular emphasis on the exact expression for the macroscopic oscillation frequency. The simulations reveal that this frequency depends not only on the detuning, but also on the effective resonance strength. The observed oscillation frequencies show excellent quantitative agreement with the relation $\Omega = \sqrt{\Delta\omega^2 + \epsilon_k^2}$, where $\Delta\omega = 2\pi(f_{\text{ac}}/f_{\text{rev}} - \nu_0)$ is the detuning and ϵ_k is the resonance strength, capturing the total effect of the AC dipole and the driven vertical betatron motion [6].

In addition to confirming the analytical form of the oscillation frequency, the simulations reveal several further effects. The polarization decay becomes more pronounced as the AC dipole frequency is tuned closer to the resonance. The oscillation amplitude itself can also decay over time, and this damping is nearly negligible under conditions of large detuning and weak AC dipole drive, but becomes appreciable when the drive frequency is very close to the resonance or when the AC dipole strength is large. Motivated by these observations, we describe the full beat phenomenon using the following function:

$$\begin{aligned} P(\text{turn}) &\approx P_0 e^{-\text{turn}/\tau_{\text{eff1}}} \\ &\quad + A\, e^{-\text{turn}/\tau_{\text{eff2}}} \cos\left(\sqrt{\Delta\omega^2 + \epsilon_k^2}\,\text{turn} + \phi\right) + B. \end{aligned} \qquad (2)$$

The oscillation frequency $\Omega = \sqrt{\Delta\omega^2 + \epsilon_k^2}$ takes the same functional form as the precession rate predicted by the Single Resonance Model (SRM) [7], which describes the spin dynamics in the vicinity of a spin–orbit resonance. This rate is an amplitude-dependent spin tune (ADST) [8] — the rate at which spins precess about the invariant spin field (ISF) $\hat{n}(\vec{z}; s)$ on a given orbital amplitude. In NRPM the resonance is not intrinsic but is driven externally by the AC dipole. The mathematical correspondence with the SRM suggests that the macroscopic polarization beat reflects an analogous structure, namely a precession of the spin about an ISF at a rate set by an SRM-like ADST.

The tracking results, however, exhibit additional features that the SRM cannot fully account for, indicating richer underlying dynamics. Hence, while the frequency expression of the proposed formula provides robust predictions, a complete description of the underlying physics will require a more comprehensive theoretical framework.

Since the macroscopic oscillation frequency carries information about the spin tune, the latter can be precisely extracted by mapping the observed oscillation frequencies across a range of constant AC dipole frequencies and performing a fit. Alternatively, because the detuning term typically dominates over the contribution from ϵ_k, the spin tune obtained from a single excitation frequency is already highly accurate in most cases. The following section presents representative simulations on the FCC-ee lattice that demonstrate the capability of NRPM for high-precision beam energy calibration in circular accelerators.

DATA ACQUISITION

Earlier simulations were carried out in Bmad [9], an established library for charged-particle dynamics with mature spin-tracking capabilities. The work has since been migrated to Xsuite [10], a modular Python framework developed at

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CERN that unifies a number of legacy accelerator tools within a single flexible environment. Spin tracking has been fully implemented and validated in Xsuite, and all results presented in this paper were produced with this framework. Adopting Xsuite enables seamless integration with the FCC-ee optics design and the existing lattice correction toolchain.

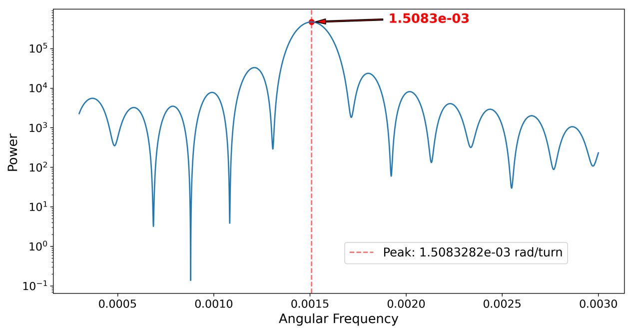

Figure 2: Example of frequency extraction using the Lomb–Scargle periodogram

The oscillation frequency was extracted in three steps. First, the high-frequency components were filtered out, and the slow decay drift was subtracted in order to suppress the dominant peak in the low-frequency region of the spectrum. A Generalized Lomb–Scargle periodogram [11] was then applied to locate the peak frequency as shown in Fig. 2. By fitting a sinusoid independently at each trial frequency, this method offers high resolution in the vicinity of the resonance and naturally handles missing data points.

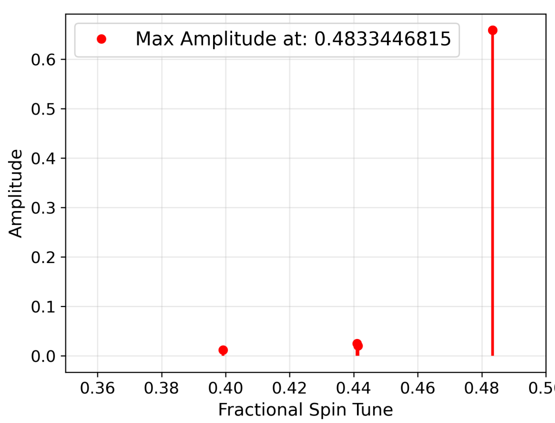

Figure 3: NAFF-based spin tune determination for the FCC-ee clean lattice

The reference spin tune is determined using the lattice without the AC dipole installed. A beam is generated with a Gaussian distribution in phase space, with the parameters provided by the Twiss module, and Monte Carlo spin tracking is performed over 10,000 turns. The complex spin signal $S_x + i\,S_z$ is then analyzed with the Numerical Analysis of Fundamental Frequencies (NAFF) algorithm [12] to extract its dominant frequency as shown in Fig. 3. The dominant peak identifies the closed-orbit spin tune ν_0. The two prominent peaks at $\nu_0 - Q_z$ and $\nu_0 - 2Q_z$, where Q_z is the synchrotron tune, arise from the modulation of the spin precession frequency by the synchrotron motion of particles. The extracted value of ν_0 is taken as the reference against which both RDP and NRPM are benchmarked.

APPLICATIONS IN THE FCC-ee

NRPM was applied to the clean FCC-ee lattice using the V105 LCC lattice at the Z energy. A 10 cm AC dipole with an integrated field of $BL = 3$ G m was installed next to an arc quadrupole at a location with $\beta_x = 48.1$ m and $\beta_y = 126.6$ m. Figure 4 shows the relation between the measured oscillation frequency Ω and the applied AC dipole frequency f_{ac}/f_{rev}. Fitting this relation yields the spin tune, which deviates from the reference value obtained via NAFF by 2.03×10^{-6}, corresponding to an energy deviation of 0.89 keV.

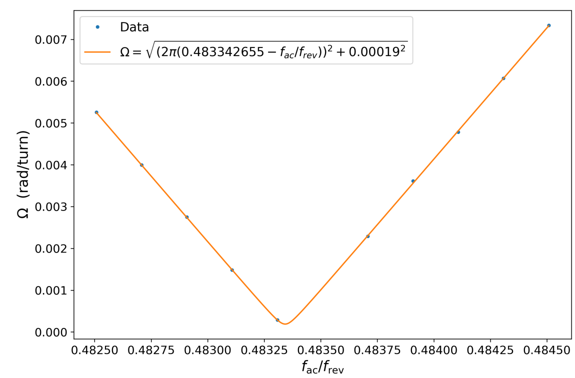

Figure 4: Fit of the oscillation frequency Ω versus the normalized AC dipole frequency f_{ac}/f_{rev} in the clean FCC-ee lattice

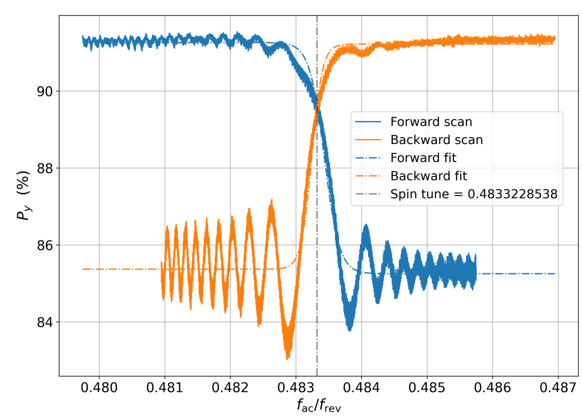

Figure 5: Simulation of forward and backward RDP scans in the clean FCC-ee lattice

For comparison, a pair of forward and backward RDP scans was performed with the same AC dipole settings, but with the frequency ramped across the resonance at 3.3 Hz/s. The resulting polarization curves in Fig. 5 exhibit the characteristic Froissart–Stora behavior associated with crossing an isolated spin resonance at finite speed [13]. Performing the scan in both directions cancels the majority of the directional bias inherent to a single sweep. The resulting polarization curves were each fitted with a logistic function, and the intersection of the two fits is taken as the resonance

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Table 1: Machine Imperfections Applied

Section	**Element**	σ_x **[µm]**	σ_y **[µm]**	σ_s **[µm]**	σ_θ **[µrad]**	**Relative field error**
ARC	Dipole	1000	1000	500	1000	**k0**: 0.001
	Quadrupole	50	50	100	50	**k1**: 0.0002
	Sextupole	50	50	100	50	**k2**: 0.0002
	Girder	150	150	500	150	N/A
STRAIGHT SECTION	IP Dipole	1000	1000	100	1000	**k0**: 0.001
	Non-IP Dipole	1000	1000	500	1000	**k0**: 0.001
	FD Quadrupole	10	10	100	10	**k1**: 0.000 01
	FD→CS Quadrupole	30	30	100	30	**k1**: 0.0001
	CS→Arc Quadrupole	100	100	100	100	**k1**: 0.0002
	Non-IP Quadrupole					
	FD→CS Sextupole	30	30	100	30	**k2**: 0.0001
	CS Sextupole					
OTHER	BPM	10	10	N/A	10	N/A

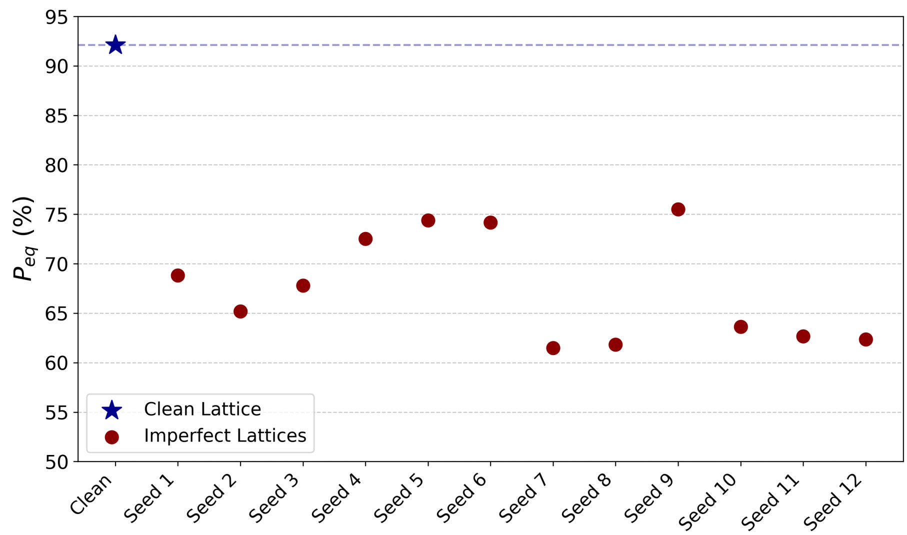

Figure 6: Equilibrium polarization level of all lattices

frequency. The obtained spin tune deviates from the reference by 2.18×10^{-5}, corresponding to an energy offset of 9.61 keV, while NRPM reaches a noticeably smaller deviation under the same simulation setting.

To assess the robustness of NRPM against machine imperfections, twelve imperfect lattices were used. They are based on the FCC-ee LCC V105 optics at the Z pole, with the imperfections listed in Table 1 applied. BPMs inherit the misalignments of their adjacent quadrupoles and are assigned a reading resolution of 10 µm. A full set of corrections is then applied, covering the orbit, phase advance, coupling, dispersion, beta-beating, tune, and chromaticity [14]. The resulting realistic and corrected lattices serves as the testbed for NRPM.

Figure 6 shows the first-order equilibrium polarization for all twelve lattices [15]. The initial spin distributions are generated such that the polarization at the start of the tracking matches the equilibrium level.

Using the same AC dipole settings as before, Fig. 7 summerizes the spin tune, ν_0, deviations from the reference value, together with the corresponding energy deviations, obtained from the forward RDP scan, the paired RDP scan, and NRPM. The paired RDP scan generally outperforms the unidirectional RDP. NRPM consistently yields a small absolute deviation across the tested lattices, demonstrating that its precision is retained under realistic lattice conditions. In addition, the RDP results tend to exhibit a systematic bias toward higher energies, whereas the NRPM results show a systematic bias toward lower energies. The origin of these opposing trends remains to be investigated.

COMPARISON OF METHODS

While NRPM could reach sub-keV precision in simulation, several practical challenges may limit its performance in real applications. The first concerns the oscillation amplitude, which must be large enough to be resolved by the polarimeter. Given the finite polarimeter resolution, an os-

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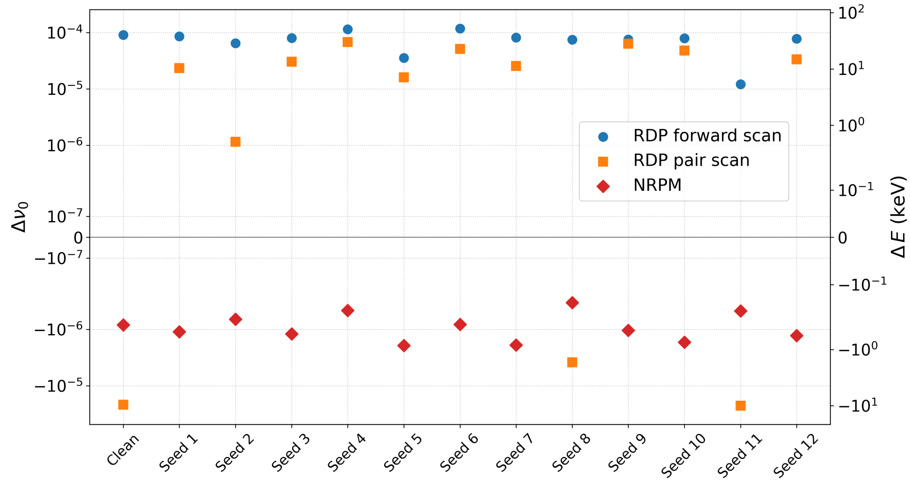

Figure 7: Measured spin tune deviation from the reference and the corresponding energy deviation across twelve imperfect lattices

cillation amplitude that is too small will be buried in the measurement noise. The second is the required sampling rate. While NRPM does not demand turn-by-turn measurement, it nevertheless places a more stringent requirement on the polarimeter sampling rate than RDP does.

Despite these challenges, NRPM offers practical benefits. For instance, it requires less time than RDP to measure the spin tune. In addition, since the AC dipole drives the spin motion coherently, the polarization undergoes a modulation rather than an irreversible decay, opening the possibility of reusing the same bunch for multiple measurements. NRPM does, however, rely on prior knowledge of the approximate location of the ν_0, and it is best paired with an injection strategy that supplies polarized beams with a high initial polarization.

RDP is well suited to rapidly localizing the spin tune over a wide range with good precision, since the sudden loss of polarization at the resonance is typically large enough to be unambiguously captured by the polarimeter. Its limitations are the longer scan times required and the reliance on a phenomenological fit to the depolarization curve, whose outcome can depend on the chosen fit function and on macroscopic features of the curve shape. NRPM, in contrast, extracts the spin tune through a frequency-domain analysis of the polarization oscillation, which isolates the signal in spectral space and is therefore less sensitive to such distortions.

A hybrid scheme combining RDP and NRPM is therefore appealing: a few coarse RDP scans can first be used to identify the approximate range of the spin tune, after which NRPM can refine the precision by exciting the beam at frequencies very close to resonance. The latter regime produces large oscillation amplitudes, easing the demands placed on the polarimeter.

CONCLUSION

A novel technique for high-precision beam energy measurement, named Near Resonance Polarization Modulation (NRPM), has been presented and validated through simulations. By driving the spin motion with a constant-frequency AC dipole in the off-resonance regime, the method induces a coherent modulation of the vertical polarization. The analytical relation between the observed modulation frequency and the detuning from the spin precession frequency allows the spin tune, and hence the beam energy, to be extracted with exceptional precision. In simulations using the FCC-ee lattice at the Z energy, NRPM is able to achieve sub-keV precision under the current simulation settings. Its effectiveness has also been confirmed in lattices that include realistic imperfections and the full correction chain.

Beyond the advantage of its precision, NRPM extracts the spin tune through a frequency-domain analysis of the polarization oscillation, making it less sensitive to the curve-level distortions than the fit-based RDP. It is also faster, and the coherent drive preserves the bulk polarization.

NRPM does place stricter demands on the polarimeter, requiring finer resolution and a higher sampling rate, and is best paired with an injection strategy supplying a high initial polarization. A hybrid RDP–NRPM scheme offers a promising path toward high-precision energy calibration in storage rings.

The underlying principles of NRPM appear broadly applicable to modern storage rings, offering a potential route to pushing the limits of energy measurement precision. Comprehensive studies are needed to fully assess NRPM's performance across a wider range of conditions, and to explore strategies for mitigating its technical challenges.

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